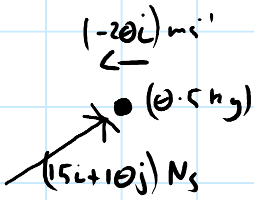
Specimen MA - M2

1)



initial momentum

$0.5 \times (-20i) = -10i$

Final momentum

$-10i + 15i + 10j = 5i + 10j$

Final Velocity

$\frac{1}{0.5}(5i + 10j) = 10i + 20j$

Final Speed $= \sqrt{10^2 + 20^2}$

$= \sqrt{500}$

$= 22.4 \text{ ms}^{-1}$ (3sf)

2)

a)

$400ms^{-1}$
$\rightarrow$
$6\times10^{-3}kg$ ●

initial KE:

$\frac{1}{2}6\times10^{-3}(400)^2 = 480J$

Final KE:

$\frac{1}{2}6\times10^{-3}(250)^2 = 187.5J$

work done by Force $= 480 - 187.5$

$= 292.5J$

$= 0.02m \times FN$

$\frac{292.5J}{0.02m} = 14,625N$

$= 14,600N$ (3sf)

3)

a) $V = \frac{d}{dt} r$

$= (3t^2 - 3)i + 8tj$

b) $(3t^2 - 3)i + 8tj = K(i+j)$

where K is constant

$3t^2 - 3 = 8t$

$3t^2 - 8t - 3 = 0$

$\frac{8 \pm \sqrt{64 - -36}}{6} = \frac{8 \pm 10}{6}$

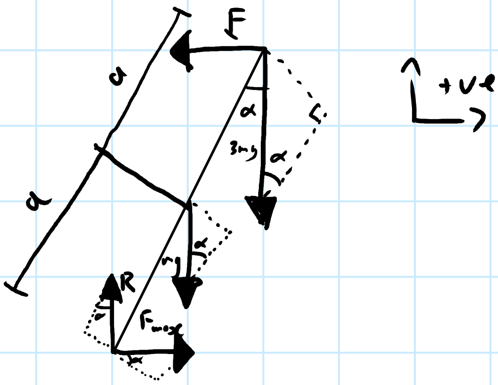
$t = 3, -\frac{1}{3}$

$t \geq 0 \therefore t \neq -\frac{1}{3}$

$t = 3$

4)

a)



we know $0^c < \alpha < \frac{\pi}{2}^c$

$0 < a$

$0 < m$

$g = 9.8$

$R - mg - 3mg = 0$

$\therefore R = 4mg$

$F_{max} = F$

$\leq \mu R$

$\leq \frac{1}{4} 4mg$

$\leq mg$

moments around centre of rod

↻ moments = ↺ moments

$aR\sin(\alpha) + a3mg\sin(\alpha) = aF_{max}\cos(\alpha) + aF\cos(\alpha)$

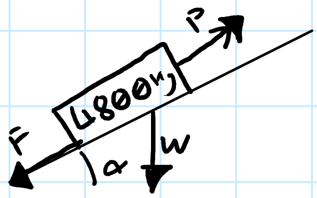
$4amg\sin(\alpha) + 3amg\sin(\alpha) \leq amg\cos(\alpha) + amg\cos(\alpha)$

$7\sin(\alpha) \leq 2\cos(\alpha)$

$\tan(\alpha) \leq \frac{2}{7}$

5)

a)



At 12ms^{-1}, 12m are travelled parallel to F in 1s

$\therefore$ Work done by F $= 2000N \times 12m$
$= 24{,}000J$

At 12ms^{-1}, 12*sin(α) m are travelled parallel to F in 1s

$\therefore$ work done by W $= 4800g \times \frac{12}{20}$
$= 28{,}224J$

$P = Et$

$P = 52{,}224J \times 1s$

$P = 52.2KW$

b)

Kinetic energy gained in 1 second = Energy from engine - Work done by F

$P = 52{,}224 - 24{,}000 = 28{,}244\,W$

$F = P \times V$ & $F = ma$ $\therefore \frac{P}{Vm} = a$

$\frac{28{,}244}{12 \times 4800} = 0.49\,ms^{-2}$

c)

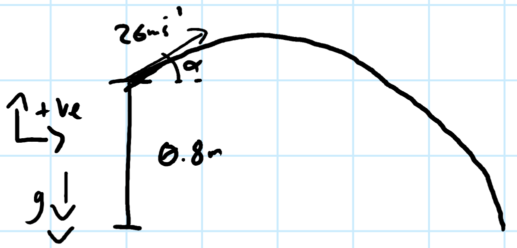
At max speed, power for the engine produces a force equal to F

$52{,}224 = 2000V$

$V = 26.112\,ms^{-1}$

$V = 26.1\,ms^{-1}$ (3 sf)

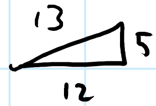
6)



$\tan(\alpha) = \frac{5}{12}$

$\sin(\alpha) = \frac{5}{13}$

$\cos(\alpha) = \frac{12}{13}$



a)

S —

U $10ms^{-1}$

V $0ms^{-1}$

A $-9.8ms^{-1}$

T X

$V^2 = u^2 + 2as$

$0 = 10^2 + -19.6S$

$19.6s = 100$

$S = 5.1m$

$h = 5.1 + 0.8$

$h = 5.9m$ (2 sf)

b) (→)

S 36m

U $24ms^{-1}$

V X

A $0ms^{-1}$

T —

$S = uT + \frac{1}{2}at^2$

$36 = 24t + \frac{1}{2}(0)t^2$

$t = \frac{36}{24}$

$t = 1.5s$

(↑)

S —

U $10ms^{-1}$

V X

A $-9.8ms^{-2}$

T 1.5s

$S = ut + \frac{1}{2}at^2$

$s = 15 + \frac{-9.8}{2} \times \frac{9}{4}$

$s = 3.975m$

$h = 4.0 + 0.8m$

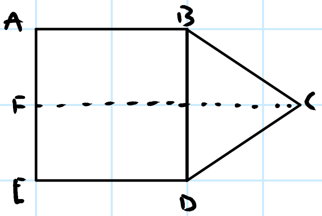
$h = 4.8m$ (2sf)

c)

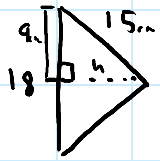
Air resistance would cause the ball to decelerate in the direction of motion

7)

Let F be the mid-point of AE so that AF = FE
The lamina is symmetrical around FC therefore COM lies on FC



Centre of mass a square lamina lies at the interception of its 2 lines of symmetry (the perpendicular bisectors of AE and AB)
Centre of mass of a triangular lamina lies 2/3 along the median line form each vertex. Since BCD is symmetrical along the perpendicular bisector of BD, then this line is the median line from C. Therefore, the COM of BCD lies 2/3 of the way towards BD, along line FC.

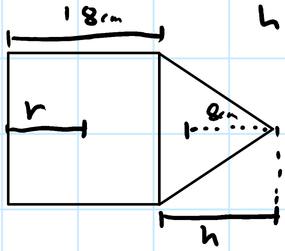


$h^2 = 15^2 - 9^2$

$= 144$

$h = 12$

$\frac{2}{3}h = 8\text{cm}$



$r = \frac{18}{2}$

$= 9\text{cm}$

area of ABDE $= 18^2$

$= 324\text{cm}^2$

area of BCD $= \frac{1}{2} 18 \times 12$

$= 108\text{cm}^2$

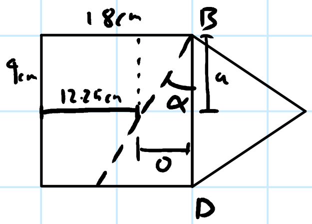
$(108+324)\bar{x} = 324 \times 9 + 108 \times [18 + (12-8)]$

$\bar{x} = \frac{2916 + 2376}{432}$

$\bar{x} = 12.25\text{cm}$

7)

b)



$o = 18 - 12.25$ $\quad a = 9\text{cm}$

$= 5.75\text{cm}$

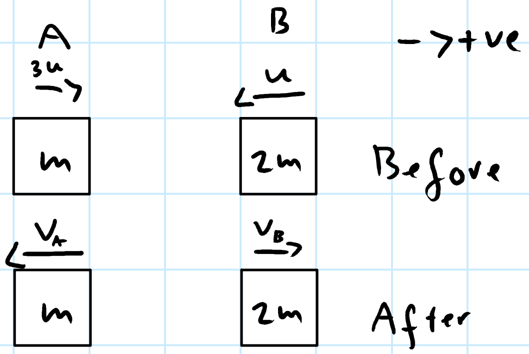
$\tan(\alpha) = \frac{o}{a}$

$= \frac{5.75}{9}$

$\alpha = 32.6^\circ$

8)

a)



$$e = \frac{V_B - -V_A}{3u - -u}$$

$$e = \frac{\text{speed of seperation}}{\text{speed of approach}}$$

$$4eu = V_B + V_A$$

momentum is conserved

$$3mu + 2m(-u) = mu$$

$$\therefore -mV_A + 2mV_B = mu$$

$$2V_B - u = V_A$$

$$4eu = V_B + 2V_B - u$$

$$4eu + u = 3V_B$$

$$V_B = \frac{1}{3}(1 + 4e)u$$

8)

b)

V_A is positive because we state it is going in the negative direction (see *)

$$2V_B - u = V_A \quad \& \quad V_A > 0$$

$$\therefore 2\left[\tfrac{1}{3}(1+4e)u\right] - u > 0$$

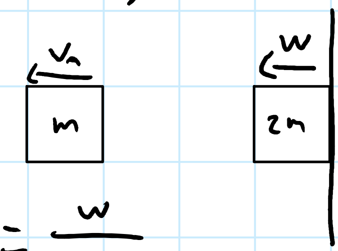
$$\tfrac{2}{3}(1+4e)u > u$$

u is positive because if it was negative, A and B would never collide
2/3 is also positive

$$1+4e > \tfrac{3}{2}$$

$$4e > \tfrac{1}{2}$$

$$e > \tfrac{1}{8}$$



c)

$$\tfrac{1}{2} = \frac{w}{V_B}$$

$$w = \tfrac{1}{6}(1+4e)u$$

For a collision to occur $w > V_A$

$$\tfrac{1}{6}(1+4e)u > \tfrac{2}{3}(1+4e)u - u$$

$$u + 4eu > 4u + 16eu - 6u$$

$$3u > 12eu$$

$$\tfrac{1}{4} > e$$